Wednesday April 3

Lecture 23

Makeup Lecture

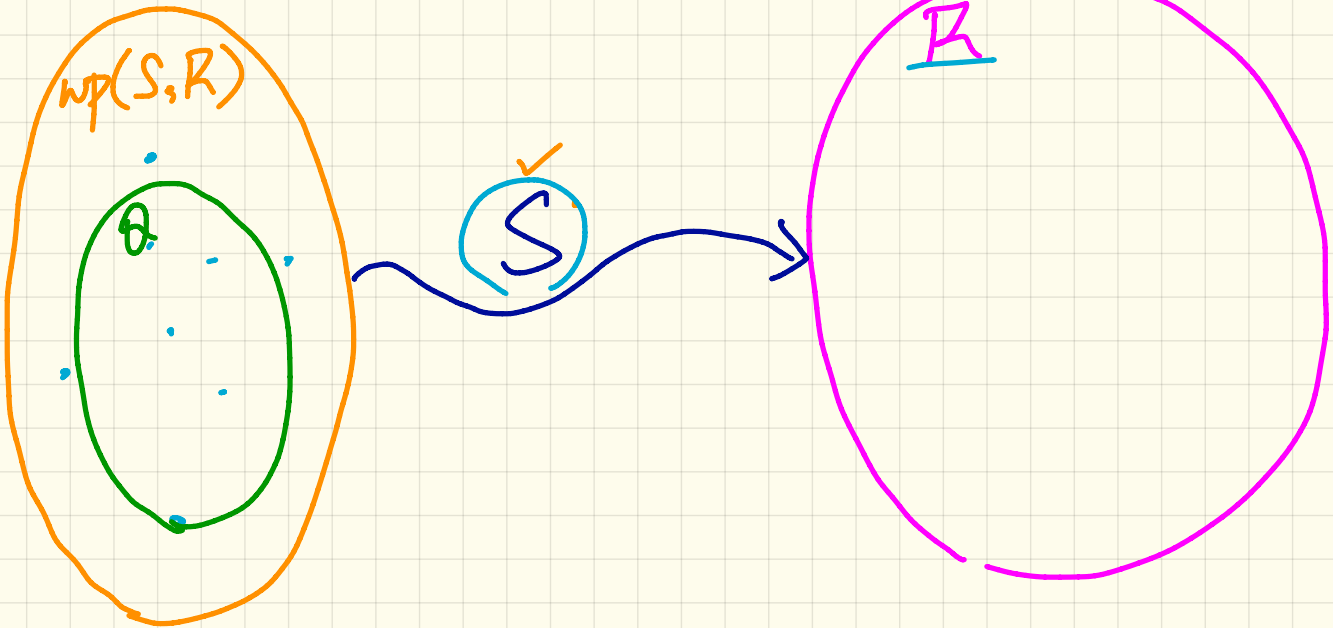
Friday April 5

2pm - 4pm

LAS B

Hoare Triple as a Predicate

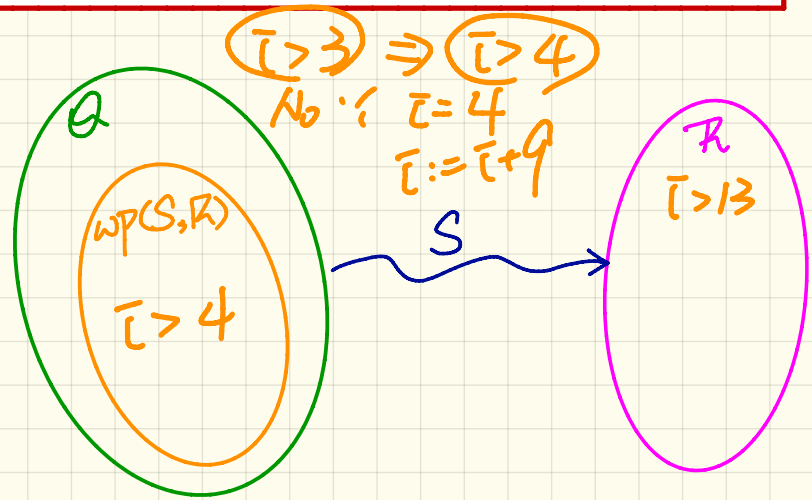
$$\{Q\}\ S\ \{R\} \equiv Q \Rightarrow wp(S, R)$$



Program Correctness : Example (1)

```
class FOO
  i: INTEGER
  increment_by_9
    require
      i > 3
    do
      i := i + 9
    ensure
      i > 13
    end
end
```

$$\{Q\}\ S\ \{R\} \equiv Q \Rightarrow wp(S, R)$$

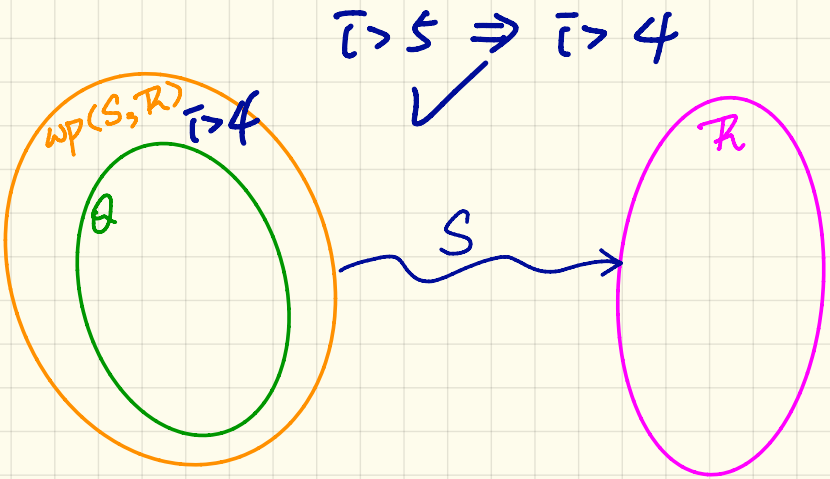


$\{i > 3\}\ \ i := i + 9\ \ \{i > 13\}$

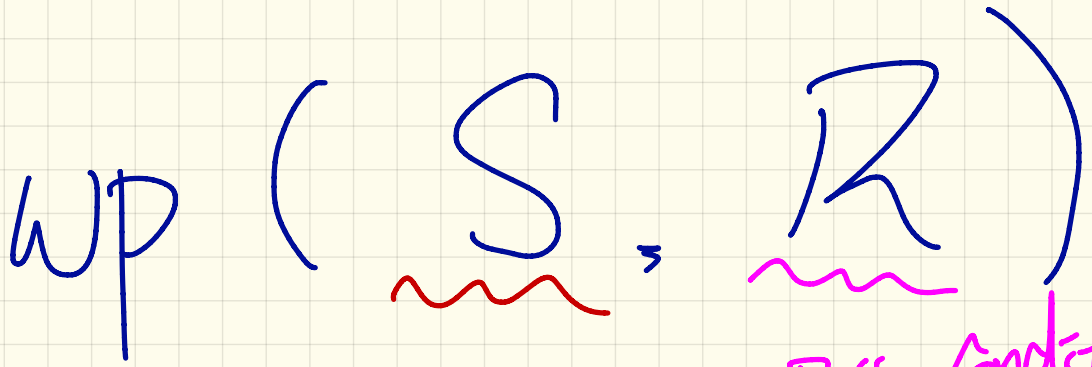
Program Correctness : Example (2)

```
class FOO
  i: INTEGER
  increment_by_9
    require
      i > 5
    do
      i := i + 9
    ensure
      i > 13
    end
end
```

$$\{Q\}\ S\ \{R\} \equiv Q \Rightarrow wp(S, R)$$



$\{i > 5\}\ \underline{i := i + 9}\ \{\underline{i > 13}\}$



Post-condition predicate

1. pre-state
2. post-state

① :=

② if then else

③ ___ ; ___

④ from · until · loop · end

Precondition

pre-state

$x_0 > 4$

$wp(x := x + 9, \underbrace{x > 13}_{\text{post-state}})$

pre-state

$x > 13$

$x_0 + 9$

$x_0 > 4$

$$wp(x := x + 1, x > x_0)$$

$$= \{ \text{wp rule for assignment} \}$$

$$x > x_0 \; [x := x_0 + 1]$$

$$= \{ \text{substitution} \}$$

$$x_0 + 1 > x_0 = \text{True}$$

any x being incremented will become larger

$$wp(x := x+1,\ x < x_0)$$

$$= \{ \ldots \}$$

$$\underline{x} < x_0 \,[\, x := \underline{x_0 + 1} \,]$$

$$x_0 + 1 < x_0$$

$$1 < 0$$

false.

When wp is true,

any precondition Q would be correct

$\because Q \Rightarrow true \equiv true$

When wp is false

only precondition false is correct but useless

$\because false \Rightarrow false \equiv true$

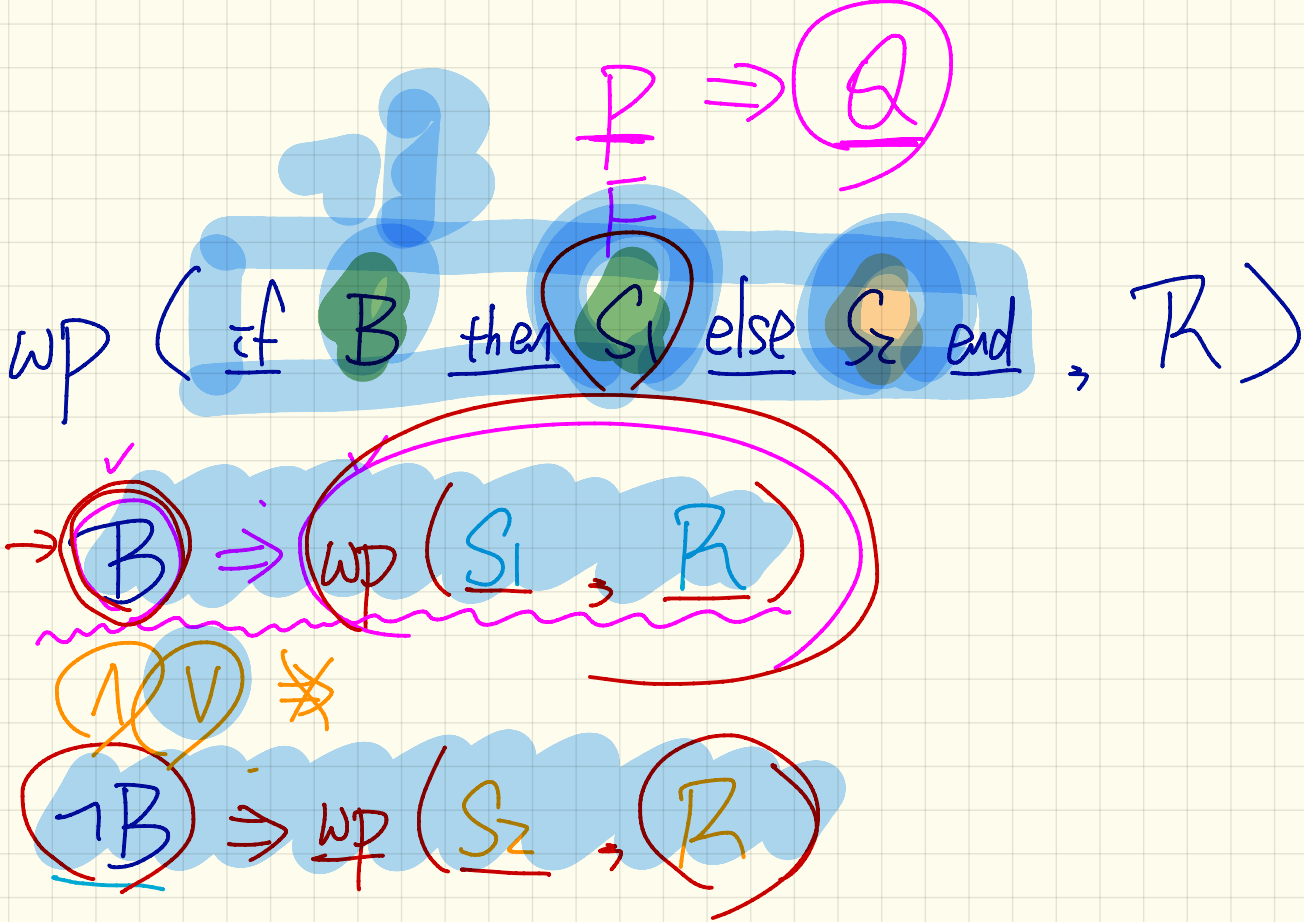
$\{x \geq 22\} \; x := x+1 \; \{x = 23\}$

$wp(x := x+1, x = 23)$

$= x = 22$

$x \geq 22 \Rightarrow x = 22$

$x = 23$


P ⇒ Q
wp (if B then S1 else S2 end , R)
B ⇒ wp (S1 , R)
¬B ⇒ wp (S2 , R)

Rule of wp: Conditionals

$wp(\text{if } B \text{ then } S_1 \text{ else } S_2 \text{ end}, R)$

$B \Rightarrow wp(S_1, R)$
$\vee$
$\neg B \Rightarrow wp(S_2, R)$

vs.

$B \Rightarrow wp(S_1, R)$
$\wedge$
$\neg B \Rightarrow wp(S_2, R)$

??

$x = -1$
$y = -1$

$x + 1 \geq 0$

Consider:

$wp(\text{if } \underbrace{y > 0}_{B} \text{ then } \underbrace{x := x + 1}_{S_1} \text{ else } \underbrace{x := x - 1}_{S_2} \text{ end}, \underbrace{x \geq 0}_{R})$

Counter example: $y = -1$, $x = -1$

$y > 0 \Rightarrow wp(x := x + 1, x \geq 0)$
$x \geq -1$

$\vee$

$y \leq 0 \Rightarrow wp(x := x - 1, x \geq 0)$
$x \geq 1$

$x = -1$
$x := x - 1$
$x = -2$

Correctness of Program: Conditionals

Is this program correct?

$\{x > 0 \wedge y > 0\}$ → B

```
if x > y then
  bigger := x ; smaller := y
else
  bigger := y ; smaller := x
end
```

$\{bigger \geq smaller\}$

S_1 S_2

$x > 0 \wedge y > 0 \Rightarrow wp$

wp

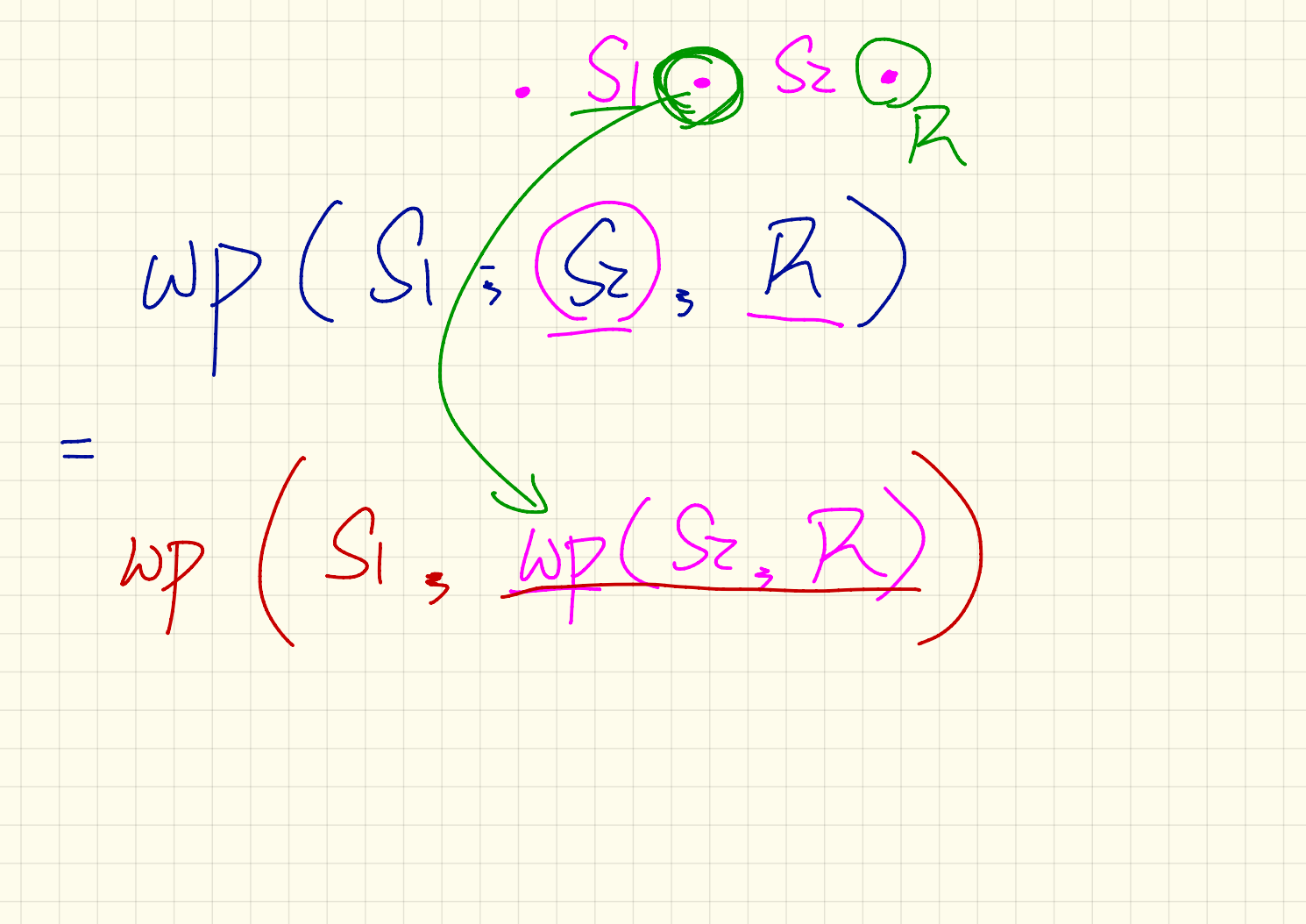
wp(if B then S_1 else S_2 end, bigger ≥ smaller)

= { wp rule for alternation }

$x > y \Rightarrow wp(S_1, bigger \geq smaller)$

$\wedge$

$x \leq y \Rightarrow wp(S_2, bigger \geq smaller)$


S1
S2
R
wp (S1 ; S2 , R)
=
wp (S1 , wp (S2 , R))